Dynamic Channel Interaction with Reference Price Effect: A Feedback Solution

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Abstract: Reference price effect describes how past price affects current demand of the frequently purchased product. In our model, we derive the feedback dynamic pricing policy under reference price effect in a decentralized channel. When the initial reference price is relatively high, the result shows that the retail price monotonically decreases firstly (initial stage), then approaches to the steady-state price (steady stage). Comparing with the integrated channel, we highlight that price penetration is more preferable than price skimming regarding to a certain group of customers. Another finding is, the channel efficiency is always lower when the consumers are more sensitive to the gap between the real market price and reference price or have less loyalty to the product. Subsequently, we investigate a myopic pricing policy and highlight that the myopic decentralized channel could coordinate the channel under some special case, resulting from the interactive effect of the myopic policy and double marginalization effect.

Keywords: Reference price effect; Feedback Pricing strategy; Channel coordination

I. Introduction

Reference price characterizes consumers' internal perception on a certain product, which is based on past price observed in prior purchasing experience (Briesch et al. [1]; Mazumdaret et al. [8]). When consumer do not have sufficient information about the product (such as quality, after sales service, etc), the reference price for the product typically deviates from the actual market price. The inconsistency between reference price and real price results in notable impacts on the consumers' purchase decision as well as sellers' market demand. A higher reference price makes consumers feel a gain and thus stimulates the demand. Conversely, a negative gap between the reference and real price leads to demand shrinkage. This kind of impact on demand is well known as reference price effect. Since the consumers' reference price typically fluctuates over time and deviates from the real price, the reference price effect is crucial to sellers' long-term profitability. Therefore, how to dynamically set the real market prices in response to the inter-temporal effect caused by reference price is clearly an essential task.

Previous researches on dynamic pricing strategy with reference price effect focus on the investigations of monopolistic and/or oligopolistic cases where the competitive effects between the downstream sellers and their suppliers are isolated from the analysis (Kopalle and Winer [7], Fibich et al. [4] and Popescu and Wu [9]). The concept of supply chain management has sparked the awareness of distribution inefficiency caused by vertical competition among channel members. A supply chain typically consists of two or more parties with vertical relationships. The independency of price decisions in a supply chain usually makes the whole channel less profitable due to the well recognized double marginalization (Spengler [18]). Double marginalization occurs when a manufacturer, as a result of selling at a wholesale price above its marginal cost, induces its intermediary to set a retail price above what it would be if it faced the true marginal cost of the channel. While the literature on supply chain management has documented a numerous mechanisms through which the incentives of the channel members can be aligned to improve the channel efficiency caused by double marginalization, most studies on this subject hold a static view of pricing decisions. Such short-term analyses do not account for the inherently dynamic nature of demand caused by reference price. There have been a number of studies examining dynamic interactions in a distribution channel (e.g., Chintagunta and Jain [3], Gutirrez and He [6]). However, no study attempts to model the reference price effect as the dynamic factor. Apparently, there is a gap in the literature concerning reference price effect in a supply chain. The objective of this paper is to close the gap by investigating the impact of reference price on the efficiency of a decentralized supply chain.

Our study is to construct a continuous Stackelberg differential game to investigate the problems mentioned above. We consider a supply chain where a manufacturer distributes a product through an independent retailer. Here the manufacturer acts as the Stackelberg leader that announces the wholesale price dynamically with no preliminary commitment (feedback strategy), while the retailer is the follower who decides the retail price. Our model yields closed-form solutions, which brings a number of insightful implications. In particular, in comparison with the vertically integrated supply chain, our results indicate that it is more probable for the retailer to adopt a price penetration strategy in the initial stage, instead of a price skimming strategy. Moreover, the channel inefficiency seems more serious in our dynamic environment. Finally, we study several alternative pricing strategies such as no reference price case and myopic case. Apparently, the no-reference price model could not reach the system optimal solution. However, a myopic decentralized channel could coordinate the channel under some special cases. We highlight the result should be ascribe to the interactive effect of myopic policy and double marginalization effect.

In summary, our research differs from the previous studies mainly in following aspects. First, for the dynamic effect on the demand side, previous researches are more focusing on durable products such as demand diffusion and saturation effect (Gutirrez and He [6]), while the reference price effect are more popular in the market of frequently purchase product (Popescu and Wu []). Then, we attempt to investigate the impacts of consumer heterogeneity on the manufacturer's choices of different pricing strategy, which have not been intensively studied yet.

The remainder of the paper is organized as follows. Related literatures are reviewed in section 2. Consequently, we will introduce how we model the reference price effect in section 3. In section 4, the centralized case is studied as a benchmark. The analysis of decentralized channel under reference price effect is given in section 5 and we also compare the profits of the centralized and decentralized channel. We propose two alternative pricing strategies in section 6 and conclude the paper in section 7.

II. The Demand Dynamics with Reference Price Effect

This section describes how reference price effect affects the process of demand dynamics. Based on the studies by Sorger [11], Kopalle and Winer [7], and Fibich et al. [4], who assume that the reference price is a weighted average of the historical price exposures of the consumer, the reference price for a product over time can be modeled by the following differential equation:

$$\dot{r}(t) = \beta(p(t) - r(t)), \tag{1}$$

where $r(t)$ is the reference price associated with an initial reference price $r(0) = r_0$ and $p(t)$ is the retail price at time t. The positive parameter β is the "memory parameter" which characterizes the memory effect. A higher β implies a shorter term memory which means less loyalty of the product. Accordingly, the demand dynamics caused by reference price effect can be specified by (Greenleaf [5] and Fibich et al.[4])

$$Q(t) = a - \delta p(t) - \gamma(p(t) - r(t)). \tag{2}$$

This formulation is derived from the classic linear demand function $Q(t) = a - \delta p(t)$, where both a, δ and γ are positive constants. Note that γ reflects the reference price effect and a higher γ implies consumers are more sensitive to the gap between the two prices. The demand will be stimulated when the real market price is higher ($r(t) > p(t)$); otherwise, it will be discouraged.

III. Centralized Channel

To establish a benchmark, we start with the centralized case where a monopolist directly sells a product to the market with the constant unit cost denoted by c. The discount rate is denoted as ρ and a higher ρ implies that one is less patient. With the demand dynamics in (2), the monopolist's net discounted profit over an infinite time horizon can be specified as

$$\Pi = \int_0^\infty e^{-\rho t}(p(t) - c)Q(t)dt. \tag{3}$$

Here the forward looking monopolist faces a price setting problem to maximize this profit. Applying the standard control theory (Sethi and Thompson [10]) , the current value Hamiltonian is given by

$$H = (p - c)[a - \delta p - \gamma(p - r)] + \lambda\beta(p - r), \tag{4}$$

where λ is the shadow price associated with the state variable $r(t)$. The shadow price can be interpreted as the impact on the future profits while increasing one unit price loss or decreasing one unit price gain. Solving the Hamiltonian, we can conclude following results.

Proposition 1 *The optimal reference price in the centralized case can be given by*

$$r(t) = M + (r_0 - M)e^{m_1 t} \tag{5}$$

and the retailer price is

$$p(t) = M + (r_0 - M)(1 + \frac{m_1}{\beta})e^{m_1 t}, \tag{6}$$

where $M = c + \frac{(\rho+\beta)(a - c\delta)}{2\delta(\rho+\beta) + \rho\gamma}$, $m_1 = \frac{\rho - \sqrt{(\rho + 2\beta)(\rho - 2n_1)}}{2}$ *and* $n_1 = -\beta\frac{\delta}{\delta+\gamma}$.

Though derived with a different approach, this result is identical to that in Fibich et al. [4]. Note that M is the optimal retailer price in the steady stage ($t \to \infty$). Since $\partial M/\partial\gamma < 0$ and $\partial M/\partial\beta > 0$, the optimal steady-state price is decreasing with the reference price effect γ, while increasing with the memory parameter β. Thus, the steady-state retail price should be higher when the customers have a lower loyalty or are less sensitive to the gap between the two prices. Moreover, the optimal retail price is always between the reference price and the steady-state price till the steady stage.

Clearly, as $m_1 < 0$, the retail price is decreasing over time when $r_0 > M$, which is called as 'price skimming'; oppositely if $r_0 < M$, the retail price is increasing with a low initial price and specified as a strategy of 'price penetration'. As r_0 represents the initial reference price of the consumers, it is apparent that the price strategies are directly impacted by their features. Furthermore, the characteristics of the monopolists themselves would also influence the choice of pricing strategies because the discount rate ρ describes the degree of their patience. It is easily verified that $\partial M / \partial \rho < 0$, which means a less patient monopolist is more likely to adopt the strategy of price skimming while facing a certain group of customers.

IV. Decentralized channel

Now we consider the decentralized case where a manufacturer distributes a product through a downstream retailer. The two channel members maximize their respective profits independently. Assume that they are both forward looking that struggle for long term profits-maximizing and the manufacturer acts as the Stackelberg leader. Moreover, no commitment is preliminarily presented by the manufacturer. To derive the equilibrium of the price setting game, we first solve the retailer's optimization problem. Let V_r and V_m denote the value functions for the retailer and the manufacturer. Based on the manufacturer's wholesale price w and discount rate ρ, the retailer's HJB equation can be specified as

$$\rho V_r = (p-w)[a-\delta p-\gamma(p-r)] + \frac{\partial V_r}{\partial r}\beta(p-r). \tag{7}$$

Based on foreseeing the retailer's reaction function, the manufacturer's HJB equation is

$$\rho V_m = (w-c)[a-\delta p-\gamma(p-r)] + \frac{\partial V_m}{\partial r}\beta(p-r). \tag{8}$$

Proposition 2 *If the manufacturer adopts a feedback pricing strategy, the optimal retail price and wholesales price are respectively*

$$p_{feedback} = R + (r_0 - R)(1+\phi/\beta)e^{\phi t} \tag{9}$$

and

$$w_{feedback} = \{\frac{3a+[3c(\delta+\gamma)+\beta\kappa]}{6(\delta+\gamma)} + [\frac{4\delta+3\gamma}{6(\delta+\gamma)} - \frac{2\phi}{3\beta}]R\} + [\frac{4\delta+3\gamma}{6(\delta+\gamma)} - \frac{2\phi}{3\beta}](r_0-R)e^{\phi t}. \tag{10}$$

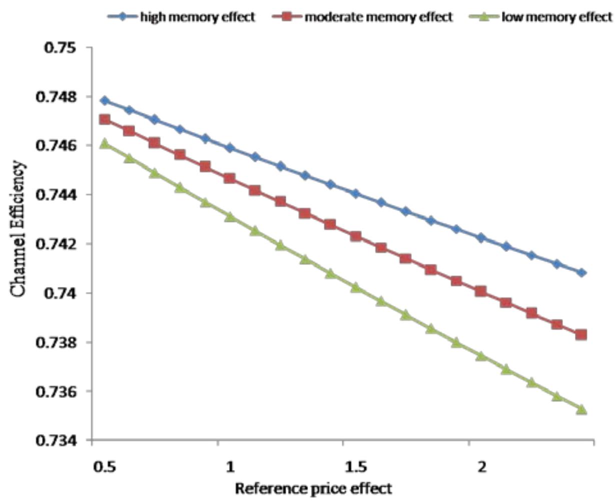
where $\phi = -\frac{4\rho(\delta+\gamma) - 4\beta\delta + \sqrt{[4\rho(\delta+\gamma)+\beta(8\delta+3\gamma)]^2 - 9\beta^2\gamma^2}}{12(\delta+\gamma)}$,

$\kappa = \frac{3[(\delta+\gamma)\phi-\delta][a-c(\delta+\gamma)] + 2a[(4\delta+\gamma)\beta - 4(\delta+\gamma)\phi]}{\beta[(2\rho+3\phi)(\delta+\gamma)-\beta\delta]}$ *and* $R = -\frac{\beta[3a+c(\delta+\gamma)+\beta\kappa]}{4(\delta+\gamma)\phi}$.

Based on Proposition 4, both the optimal retail price and wholesale price are totally deviated from the open-loop case. For the manufacturer, if she adopts a feedback pricing strategy, the discount rate constantly affects her decision. Moreover, in the steady stage, the feedback wholesale price is still impacted by the reference price effect and memory effect. It could be directly verified that $\phi < 0$, as a result, when $r_0 > R$, the retail price is decreasing over the planning horizon and vice verse.

To investigate the impacts of reference price effect on the total discounted profit of the channel, we employ the numerical methods in which the parameters are similar to Fibich et al [4] ($r_0 = 5$, $\rho = 0.05$, $c = 1$, $a = 10$ and $\delta = 2$). Following figure shows the channel efficiency under different memory effects (β) and reference price effects (γ). Here we assume $\beta = 0.5,\ 1,\ 2$ respectively represent the low, moderate and high memory effect.

Fig: The Impacts of Reference Effect and Memory Effect on Channel Efficiency



Here channel efficiency is positive to memory effect and negative to reference effect. Therefore, when the customers are more sensitive to the reference price or have a stronger loyalty, the channel efficiency will be lower and the problem of double marginalization is more serious. This result is contrary with the steady stage which means mitigating the double marginalization in steady stage could diminish the channel efficiency over the whole planning horizon. Moreover, as channel efficiency in traditional static channel is commonly 3/4 (Chiang et al.[2]), double marginalization in this inter-temporal framework is more critical. Similar conclusions could be observed under other dynamic factors such as cost learning effect.

V. Alternative Pricing Strategies

In this section, we will compare several different pricing strategy adopted by the manufacturer and the retailer. Firstly, the case of no-reference price model is investigated that could be a benchmark. Moreover, a greedy pricing strategy will be employed by the parties of the channel when they pursuit for the current profit-maximization.

No-Reference price Model

When the reference price effect is absent, the problem is consistent with the classic demand-supply model. Therefore, the objective functions of the manufacturer and the retailer are respectively $\pi_m^{NR} = (p^{NR} - w^{NR})(a - \delta p^{NR})$ and $\pi_R^{NR} = (w^{NR} - c)(a - \delta p^{NR})$. Clearly, the optimal retail price $P^{NR} = (3a + \delta c)/4\delta$ and wholesale price $w^{NR} = (a + \delta c)/2\delta$. As widely discussed, the decentralized channel could not reach the system-optimal solution.

Myopic case

Then we consider an alternative pricing strategy that both the retailer and manufacturer maximize their instantaneous profits, which is specified as a 'myopic pricing strategy'. Here their objective functions are

$$\pi^r = (p - w)[a - (\delta + \gamma)p + \gamma r] \quad (11)$$

and

$$\pi^m = (w - c)[a - (\delta + \gamma)p + \gamma r]. \quad (12)$$

Proposition 3 *When the manufacturer and the retailer adopt a myopic pricing policy, the optimal wholesales price and retail price can be given by*

$$w_{myopic} = \frac{(2\delta + \gamma)c + 2a}{4\delta + \gamma} + \frac{\gamma}{2(\delta + \gamma)}[r_0 - \frac{c(\delta + \gamma) + 3a}{4\delta + \gamma}]e^{-\frac{4\delta + \gamma}{4(\delta + \gamma)}\beta t} \quad (16)$$

and

$$p_{myopic} = \frac{(\delta + \gamma)c + 3a}{4\delta + \gamma} + \frac{3\gamma}{4(\delta + \gamma)}[r_0 - \frac{c(\delta + \gamma) + 3a}{4\delta + \gamma}]e^{-\frac{4\delta + \gamma}{4(\delta + \gamma)}\beta t} \quad (17)$$

Hence, the myopic wholesales and retail price in the steady stage is $w_{myopic}^{ss} = \frac{(2\delta + \gamma)c + 2a}{4\delta + \gamma}$ and $p_{myopic}^{ss} = \frac{(\delta + \gamma)c + 3a}{4\delta + \gamma}$. In steady stage, both the myopic wholesales price w_{myopic}^{ss} and retailer price p_{myopic}^{ss} decrease in γ. Subsequently, we studied the channel profit in the steady stage and obtain following proposition.

Proposition 4 In the steady stage, the myopic decentralized channel could coordinate the channel only when $\gamma = 2\delta$.

As discussed in the Fibich et al.[4], the myopic centralized channel could not reach the system optimal solution in the present of reference price effect. This result could be observed as a result of combination effect from the myopic pricing strategy and the double marginalization effect. The myopic policy makes the retail underprice the product (Popescu and Wu [9]), while the double marginalization effect results in a higher retail price.

VI. Discussions

In this study we present a dynamic game to investigate competitive pricing strategies in a distribution channel in the presence of reference price effect. We derive feedback equilibrium of the dynamic pricing game conclude that:

(1) The optimal steady-state retail price is higher in the decentralized channel due to the double marginalization effect. Therefore, for certain group of consumers, it is more likely for the retailer to adopt a pricing penetration strategy under a disintegrated channel.

(2) When the customers are more sensitive to the reference price or have a stronger loyalty, the channel efficiency will be lower and the problem of double marginalization is more serious.

(3) The inter-temporal decentralized channel always suffers from a more serious double marginalization effect. This result is not only tenable in our model of reference price effect. Previous studies show similar results in other dynamic factors such as cost learning effect.

Subsequently, two types of alternative pricing strategies are introduced: the no-reference price model and the myopic pricing strategy. The results illustrates that the price patterns of myopic strategy is similar to the open-loop case, which is, it also make a choice of pricing skimming or price penetration due to the consumers' features. However, although the myopic integrate channel could not reach the optimal system state; the myopic decentralized channel could coordinate the channel under some special cases. We believe it results from the interactive effect of myopic policy and double marginalization effect.

There are several foreseeable extensions for our work. First, our model is based on a simple channel structure of a single manufacturer and retailer. In future it is interesting to study the dynamic pricing problems under a more complex supply chain, e.g. multiple manufactures and retailers. Under this condition, both the vertical and horizon competition should be faced. Moreover, the channel members could choose either open-loop or feedback pricing strategy. This problem should be very valuable but tough at the same time. Also, as discussed in the static channel, some channel contracts such as revenue sharing are applied to coordinate the decentralized channel by mitigating the double marginalization. It's interesting to study this problem in our inter-temporal framework.

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